

TMU 2020 PILOT
Teaching Mathematics for Understanding
GRADE 4
TERM 1
TEACHER TRAINING HANDOUTS

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Materials Distribution Form

Please take time to:

- check off each item as you receive it.
- write your name on each item that you are given.
- keep all your materials together.

Remember to look after these resources.

For example:

- a. Try to stick flashcards on cardboard, and laminate them.
- b. Store lesson plans carefully, so they do not fall apart and get ruined.

If you leave the school or change grades, please pass the relevant resources on to the next teacher.

2020TERM	GRADE	ITEM	CHECK
1	4	LESSON PLAN	
1	4	TEACHER RESOURCE PACK	
1	4	LEARNER ACTIVITY BOOK (LAB)	
1	4	REVISION PROGRAMME	
1	1-3	BILINGUAL DICTIONARY	

PILOT participation

CURRENT EXPERIENCE WITH GRADE 4 MATHEMATICS TEACHING

CONCERNS ABOUT THE TMU PILOT

WHAT I HOPE TO GET OUT OF THE TMU PILOT

A large, empty rectangular box with a thin black border, intended for the user to write their responses to the question 'WHAT I HOPE TO GET OUT OF THE TMU PILOT'. The box occupies most of the upper half of the page.

Lesson plan adaptation to the Mathematics Framework

General Adaptation to align with the Mathematics Framework

- The framework calls for **Teaching Mathematics for Understanding (TMU)**
- The framework suggests that there should be a balance of conceptual, procedural, strategies and reasoning, in a learning centred classroom.

The framework dimensions were well represented in the lesson plans you have already been provided with – but the balance of the dimensions can always be improved. The new lesson plans include improved and changed activities, more scaffolding in some activities and more notes to help teachers teach in a way that is aligned to the Mathematics Framework.

The framework also suggests that the vertical algorithm should be one of the algorithms that teachers are allowed to teach. This was not present in the previous lesson plans as it was not explicitly in the CAPS. This is a significant change in both the Foundation Phase and the Intermediate Phase. All algorithms should be taught by building on concepts. But alternative algorithms are not left out – they are all part of the curriculum.

The framework also calls for a reorganisation of the CAPS in order to allow for greater depth of teaching. Thus the sequencing of the lessons has changed. The sequencing of the reorganised CAPS has been developed for the TMU PILOT by JICA at the DBE. This curriculum strengthening is a critical element of the TMU PILOT which has come about as a result of the framework.

TMU PILOT materials

These consist of:

1. Lesson Plans with unit planner– including revision lessons for the first two weeks.
2. A teacher resource pack (lesson handouts, flashcards and assessment tasks).
3. Learner Activity Books (daily classwork and homework activities – LAB).
4. Bilingual dictionaries (composite Grade 1-2-3 word list with explanations, examples and illustrations).

Adaptations were made to these components of the lesson plans

- Lesson plan outline (see adaptations in Lesson Plan document front pages)
- Design of lesson plan:
 - UNIT Introduction provided per unit
 - **Links to the Mathematics Framework – Teaching Mathematics for Understanding.**
 - Fully planned lessons
 - Consolidation lessons
 - Assessment Tasks
 - Printable versions of assessments provided in the Teacher Resources.
- **Timing of lesson components (adapted):** For best use of the 60-minute lesson – the following was agreed.
 - Mental maths (5 minutes)
 - Link to previous lesson (5 minutes)
 - Homework correction (5 minutes)
 - Lesson content – concept development and Classwork activity (35 minutes)
 - Homework activity (5 minutes)

- Reflection and summary of lesson (5 minutes)
- Simplified Unit Plan and Overview – This provides opportunities to keep a record of work and to reflect on work done.

TMU PILOT Methodology

Conceptual understanding is one of the most critical ideas that teachers have to build in learners. The content input in this training session is intended to introduce the following methodologies that contribute to the conceptual understanding of mathematics.

1. CPA approach
2. Base ten number system with place value table
3. Word problems with diagrams
4. Addition and subtraction in column

‘CPA approach’ stands for a **concrete-pictorial-abstract** approach. It helps Grade 4 learners to make connections between concrete objects to number symbols and number sentences. This is a key for bringing up number sense. It is not a linear process – learners go between the three levels of activities forwards and backwards, sometimes many times, before the final consolidation of abstract understanding.

Using diagrams in word problem helps learners understand the question and work out an operation to find the solution to the problem.

See **Lesson Plan** inserts for details on the TMU PILOT methodology and vocabulary.

Lesson Demonstrations

- Take note of the lesson that has been assigned to you for demonstration.
- Read the lesson plan (not only the lesson you demonstrate, but also the previous and the following lesson plans to see the progression) and the core methodology to help you prepare your lesson.
- Remember to pay close attention to your time management and incorporate good classroom management strategies.
- Do not do too much speaking – give concise, clear instructions and explanations.
- Take care to use the concrete materials in the appropriate way and give time to learners to manipulate the materials as well. Allow the learners time to explain what they are doing.
- Give time to learners to write down their work in their classwork books as well, especially since, it is the first time for many of Grade 4 learners to learn mathematics in English.

Unit	Lesson no	Lesson content	Time Allocation	Presenters
1	5	Order, Compare and represent numbers	25 minutes	
1	10	Dividing by 10, 100, 1 000	25 minutes	
2	17	Using the column method to subtract 5-digit numbers	25 minutes	
2	20	Using other methods to subtract big numbers	25 minutes	
3	28	Using rounded numbers	25 minutes	
3	32	Approximation by grouping	25 minutes	
4	40	Geometric patterns, tables and flow diagrams (1)	25 minutes	
4	46	More number sentences (2)	25 minutes	

Teaching in multilingual contexts

Please discuss the following questions. Think about examples from the classroom that are relevant as well.

1. Do you feel comfortable teaching mathematics in English?
2. What is your view on the language policy for Grade 4?
3. What do you think the LoLT for mathematics should be? From Grade 1 (HL/English?)
4. What do you do when you see that the learners don't understand what you are saying? (explain/use concrete material/other?)
5. Have you noticed language inconsistencies in textbooks/DBE workbook/CAPS?
6. Did your learners experience any problems with regard to vocabulary when the learners wrote the ANA? (specific examples/general).
7. How do you use code-switching or translanguaging strategies in your teaching?

Classroom Management

CREATING A POSITIVE LEARNING ATMOSPHERE

The best learning takes place when learners feel safe and confident enough to participate. It is up to you, as the teacher, to create the kind of atmosphere that will promote discussion and learning. Below are some tips to help you with this important challenge:

1. **Work constantly to create the atmosphere that you want.** It takes time for teachers and learners to build relationships, and to adopt the behaviours required for a safe, positive classroom. Don't give up if it doesn't happen straight away – keep working towards creating a feeling of emotional safety in your classroom.
 - ***How can you behave differently to try and make your learners feel safe and confident in your classroom?***

2. **Take an interest in learners' work.** Most of the time, you will probably get learners to correct their own work. However, it is a good idea to look through learners' exercise books regularly. This allows you to verify that your learners are doing their work and are on track. It is also a time for you to show interest in learners' progress. Tell learners when you are pleased or impressed with their progress or efforts. This shows learners that you are interested, supportive, and that you value their work. Also, display learners' work in the classroom on a regular basis – change these displays often and give many learners the opportunity to display their work. This makes learners feel proud, and it also gives them a sense of ownership of the classroom.
 - ***What kinds of things do you notice when you mark your learners' books?***
 - ***Is this helpful to you? Why or why not?***
 - ***How can you display learners work? What practical solutions can you suggest?***

3. **Establish and implement rules.** Work out a set of rules for your classroom – it is a good idea to do this together with the learners.
 - a. Explain that learners must think of their classroom as their 'home' at school. Just like they have rules at home, we need rules at school. These rules help us to live happily and safely together.
 - b. Ask learners to contribute their ideas for the rules.
 - c. Try to phrase these rules in the positive. For instance, say: **Walk quietly in class**, rather than: **Do not run**.
 - d. When everyone has contributed their ideas, add your own ideas, and finalise your list.
 - e. **Explain the reasons for each rule to the learners. Let them know that the rules are there to make their 'home' at school pleasant and safe.**
 - f. Finally, ask learners if they are all prepared to accept and live by these rules. If there is a rule that needs to be adjusted or removed, do so.
 - g. Get every learner to sign the rules – the Grade 4 learners should be able to write their name or initial at the bottom of the rules.
 - h. Learners need to know that you will take action against harmful behaviour. If you do not do this, it will be difficult for learners to trust you.
 - ***What kind of rules do you think are appropriate and useful in an Intermediate Phase classroom?***
 - ***Will these rules make learners feel safe and confident? If so, how?***

4. **Correct mistakes clearly, but without making the learner feel bad.** When learners make mistakes, thank them for trying, but point out that a mistake has been made. Correct the mistake clearly and quickly, and then move on. Do not labour the point – learners must see that it is perfectly acceptable to make mistakes as long as one tries.
 - ***How does it feel not to know if your answer is correct or incorrect?***
 - ***How can you correct a learners' mistake without making him or her feel bad?***

5. **Tell learners if you do not know something.** Learners appreciate it when teachers are honest, and say things like, 'I'm not really sure. Does anyone else know? Should we look up the answer?'
 - ***What do you think it shows the learners when you don't know the answer?***
 - ***What do you think learners can learn from this?***

6. **Model the kind of behaviour you expect in your class.** We often hear the phrase 'respect is earned', or 'respect is a two-way street', but we don't always think about what that means.
 - a. The simplest explanation is to model the behaviour that you expect from your learners, and to treat them the way that you want to be treated.
 - b. Be punctual and prepared for lessons; work diligently; keep your space clean, tidy and organised; never use your cell phone in class; look after your apparatus and resources; greet learners; be considerate of their feelings; praise learners for a job well done; thank learners for going the extra mile; and go the extra mile yourself.
 - c. This may not be reciprocated immediately, but in time, learners will learn from your model, and will begin to behave as you do within your environment.
 - d. Feel free to hold an open, honest discussion with learners about this concept. Let learners know that you will try to always treat them with consideration and respect, and that you will always work hard for them.
 - e. Let your learners know that you will appreciate them trying to do the same.
 - ***Is this something that you have been doing, or is this a new concept to you?***
 - ***Does it make sense, that we should treat learners the way that we want them to treat us?***
 - ***What are some ideas of things that you can do differently?***

7. **Move around the classroom.** As learners work, walk around the classroom. Use this opportunity to stop and look at individual learner's work. Stop and discuss challenges – help individual learners as much as you can. Look out for problems between learners, and deal with issues that arise. Get to know your learners better. If tension is building between learners, put a stop to the argument. Then, find time for the learners to talk it out while you mediate.
 - ***Can you move freely around your classroom?***
 - ***If not, can you change the way your desks are arranged to help you move around?***
 - ***Is there anything that you can move out of your classroom to make more space?***

8. **Laugh with your learners.** If you can find something to laugh about with your learners, do so! This is an excellent way to bond with learners, and to make them feel closer to you. Laughter is also an excellent way to break down tensions, and to get learners to relax.
 - ***What kinds of things do you and your learners laugh about?***
 - ***How do you explain to learners that whilst it is good to laugh together, we must never laugh at someone?***

9. **Leave your problems outside of the classroom.** Learners pick up on your stress, anxiety and unhappiness, and this can affect them negatively. Try your best to be in the habit of leaving your problems at the classroom door, and to focus on your learners once you are inside the classroom.
 - ***Have you got any strategies to help you do this? If so, what are they?***

10. **Praise your learners for their efforts.** This is one of the easiest and most effective behaviours that you can implement.
 - a. Praise learners not for their achievements, but for their efforts. This will encourage learners to try and do more.
 - b. This is known as building a 'growth mindset'. This means that learners believe that they can learn and progress.
 - c. The opposite of a growth mindset is a 'fixed mindset', where learners believe they are born with a certain ability, and that they cannot change this.
 - ***What were you told about yourself as a child, that made you believe that you were not good at something?***
 - ***Did this affect your attitude? Did you try to do this thing and get better at it, or not?***

Specification of Content (Grade overview)

NUMBERS, OPERATIONS AND RELATIONSHIP			
TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
Whole numbers	<p>Mental calculations involving:</p> <ul style="list-style-type: none"> Addition and subtraction of: <ul style="list-style-type: none"> – units – multiples of 10 – multiples of 100 – multiples of 1 000 – multiples of 10 000 – multiples of 100 000 Multiplication of whole numbers to at least 10 x 10 Multiplication facts of: <ul style="list-style-type: none"> – units by multiples of 10 – units by multiples of 100 – units by multiples of 1000 – units by multiples of 10 000 – units by multiples of 100 000 <p>Number range for counting, ordering, comparing and representing, and place value of digits</p> <ul style="list-style-type: none"> Count forwards and backwards in 2s, 3s, 5s, 10s, 25s, 50s, 100s, 1 000s, 10 000s and 100 000s between 0 and at least 1 000 000. Order, compare and represent numbers to at least 6-digit numbers Represent odd and even numbers to at least 1 000 000. Recognize the place value of digits in whole numbers to at least 6-digit numbers Round off to the nearest 10, 100, 1 000, 10 000, 100 000 	<p>Mental calculations involving:</p> <ul style="list-style-type: none"> Addition and subtraction of up to multiples of 1000: Multiplication facts of units by multiples up to 1 000: <p>Number range for counting, ordering, comparing and representing, and place value of digits</p> <ul style="list-style-type: none"> Count forwards and backwards in 2s, 3s, 5s, 10s, 25s, 50s, 100s, between 0 and at least 10 000 Order, compare and represent numbers to at least 4-digit numbers Represent odd and even numbers to at least 1 000. Recognize the place value of digits in whole numbers to at least 4-digit numbers Round off to the nearest 10, 100, 1 000 	<p>Mental calculations involving:</p> <ul style="list-style-type: none"> Addition and subtraction of: <ul style="list-style-type: none"> – multiples of 10 000 – multiples of 100 000 Multiplication facts of: <ul style="list-style-type: none"> – units by multiples of 10 000 – units by multiples of 100 000 <p>Number range for counting, ordering, comparing and representing, and place value of digits</p> <ul style="list-style-type: none"> Count forwards and backwards in 1 000s, 10 000s and 100 000s between 0 and at least 1 000 000. Order, compare and represent numbers to at least 6-digit numbers Represent odd and even numbers to at least 1 000 000. Recognize the place value of digits in whole numbers to at least 6-digit numbers Round off to the nearest 10 000, 100 000

	<p>Number range for calculations</p> <ul style="list-style-type: none"> • Addition and subtraction of whole numbers of at least 6 digits • Multiplication of at least whole 2-digit by 2-digit numbers • Division of at least whole 3-digit by 1-digit numbers <p>Calculation techniques</p> <ul style="list-style-type: none"> • Use a range of techniques to perform and check written and mental calculations of whole numbers including: <ul style="list-style-type: none"> – estimation – adding, subtracting and multiplying in columns – long division – building up and breaking down numbers – rounding off and compensating – doubling and halving – using a number line – using addition and subtraction as inverse operations – using multiplication and division as inverse operations 	<p>Number range for calculations</p> <ul style="list-style-type: none"> • Addition and subtraction of whole numbers of at least 4 digits • Multiplication of at least whole 2-digit by 2-digit numbers • Division of at least whole 3-digit by 1-digit numbers <p>Calculation techniques</p> <ul style="list-style-type: none"> • Use a range of techniques to perform and check written and mental calculations of whole numbers including: <ul style="list-style-type: none"> – estimation – building up and breaking down numbers – rounding off and compensating – doubling and halving – using a number line – using addition and subtraction as inverse operations – using multiplication and division as inverse operations 	<p>Number range for calculations</p> <ul style="list-style-type: none"> • Addition and subtraction of whole numbers of at least 6 digits <p>Calculation techniques</p> <ul style="list-style-type: none"> • Use a range of techniques to perform and check written and mental calculations of whole numbers including: <ul style="list-style-type: none"> – Adding, subtracting and multiplying in columns – long division
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TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
Whole numbers	<p>Number range for multiples and factors</p> <ul style="list-style-type: none"> • Multiples of 1-digit numbers to at least 100 <p>Properties of whole numbers</p> <ul style="list-style-type: none"> • Recognize and use the commutative, associative, and distributive properties with whole numbers • 0 in terms of its additive property • 1 in terms of its multiplicative property <p>Solving problems</p> <ul style="list-style-type: none"> • Solve problems in contexts involving whole numbers, including <ul style="list-style-type: none"> – financial contexts – measurement contexts • Solve problems involving whole numbers, including <ul style="list-style-type: none"> – comparing two or more quantities of the same kind (ratio) – comparing two quantities of different kinds (rate) – grouping and equal sharing with remainders 	<p>Number range for multiples and factors</p> <ul style="list-style-type: none"> • Multiples of 1-digit numbers to at least 100 <p>Properties of whole numbers</p> <ul style="list-style-type: none"> • Recognize and use the commutative, associative, and distributive properties with whole numbers <p>Solving problems</p> <ul style="list-style-type: none"> • Solve problems in contexts involving whole numbers, including <ul style="list-style-type: none"> – financial contexts – measurement contexts • Solve problems involving whole numbers, including <ul style="list-style-type: none"> – comparing two or more quantities of the same kind (ratio) – comparing two quantities of different kinds (rate) – grouping and equal sharing with remainders 	<p>Properties of whole numbers</p> <ul style="list-style-type: none"> • 0 in terms of its additive property (was lifted in clarification notes) • 1 in terms of its multiplicative property
Common fractions	<p>Describing and ordering fractions:</p> <ul style="list-style-type: none"> • Compare and order common fractions and mixed numbers (halves; thirds, quarters; fifths; sixths; sevenths; eighths; tenths), where a denominator is a multiple of another. • Describe and compare common fractions in diagram form <p>Calculations with fractions:</p> <ul style="list-style-type: none"> • Addition and subtraction of common fractions and mixed numbers with the same denominators • Recognize, describe and use the equivalence of division and fractions <p>Solving problems</p> <ul style="list-style-type: none"> • Solve problems in contexts involving fractions, 	<p>Describing and ordering fractions:</p> <ul style="list-style-type: none"> • Compare and order common with different denominators (halves; thirds, quarters; fifths; sixths; sevenths; eighths; tenths) • Describe and compare common fractions in diagram form <p>Calculations with fractions:</p> <ul style="list-style-type: none"> • Addition of common fractions with the same denominators • Recognize, describe and use the equivalence of division and fractions <p>Solving problems</p> <ul style="list-style-type: none"> • Solve problems in contexts involving fractions, 	<p>Describing and ordering fractions:</p> <ul style="list-style-type: none"> • Compare and order mixed numbers (tenths), where a denominator is a multiple of another. <p>Calculations with fractions:</p> <ul style="list-style-type: none"> • Addition and subtraction of mixed numbers with the same denominators

	<p>including grouping and equal sharing</p> <p>Equivalent forms:</p> <ul style="list-style-type: none"> Recognize and use equivalent forms of common fractions (fractions in which one denominator is a multiple of another) 	<p>including grouping and equal sharing</p> <p>Equivalent forms:</p> <p>Recognize and use equivalent forms of common fractions (fractions in which one denominator is a multiple of another)</p>	<p>Equivalent forms:</p> <p>N/A</p>
Decimal fractions	<p>Recognizing, ordering and place value of decimal fractions</p> <ul style="list-style-type: none"> Count forwards and backwards in decimal fractions to at least one decimal place Compare and order decimal fractions to at least one decimal place Place value of digits to at least one decimal place <p>Calculations with decimal fractions</p> <ul style="list-style-type: none"> Addition and subtraction of decimal fractions with at least one decimal place Multiply decimal fractions by 10 <p>Solving problems</p> <ul style="list-style-type: none"> Solve problems in context involving decimal fractions <p>Equivalent forms:</p> <ul style="list-style-type: none"> Recognize equivalence between common fraction and decimal fraction forms of the same number where the denominator is a factor of 10 	N/A	Everything on decimals

PATTERNS, FUNCTIONS AND ALGEBRA

TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
Numeric patterns	<p>Investigate and extend Numeric patterns</p> <ul style="list-style-type: none"> • Investigate and extend numeric patterns <ul style="list-style-type: none"> – involving a constant difference – of learners own creation • Describe observed relationship or rules in own words or by number sentence <p>Input and output values</p> <ul style="list-style-type: none"> • Determine the input and output values given rules in <ul style="list-style-type: none"> – flow diagram – tables • Determine the rule given input and output values in flow diagrams <p>Equivalent forms</p> <ul style="list-style-type: none"> • Determine equivalence of different descriptions of the same relationship or rule presented: <ul style="list-style-type: none"> – verbally – in a flow diagram – in a table – by a number sentence 	<p>Investigate and extend Numeric patterns</p> <ul style="list-style-type: none"> • Investigate and extend numeric patterns <ul style="list-style-type: none"> – involving a constant difference – of learners own creation • Describe observed relationship or rules in own words or by number sentence <p>Input and output values</p> <ul style="list-style-type: none"> • Determine the input and output values given rules in <ul style="list-style-type: none"> – flow diagram – tables • Determine the rule given input and output values in flow diagrams <p>Equivalent forms</p> <ul style="list-style-type: none"> • Determine equivalence of different descriptions of the same relationship or rule presented: <ul style="list-style-type: none"> – verbally – in a flow diagram – in a table – by a number sentence 	N/A

<p>Geometric patterns</p>	<p>Investigate and extend geometric patterns</p> <ul style="list-style-type: none"> Investigate and extend numeric patterns <ul style="list-style-type: none"> – represented in physical or diagram form – not limited to a constant difference – of learners own creation Describe observed relationship of the above patterns in own words Describe observed rules of patterns involving constant difference in own words or by number sentence <p>Input and output values</p> <ul style="list-style-type: none"> Determine the input and output values given rules in <ul style="list-style-type: none"> – flow diagram – tables Determine the rule given input and output values in flow diagrams and tables <p>Equivalent forms</p> <ul style="list-style-type: none"> Determine equivalence of different descriptions of the <u>same relationship or rule</u> presented <ul style="list-style-type: none"> – verbally – in a flow diagram – in a table – by a number sentence 	<p>Investigate and extend geometric patterns</p> <ul style="list-style-type: none"> Investigate and extend numeric patterns <ul style="list-style-type: none"> – represented in physical or diagram form – not limited to a constant difference – of learners own creation Describe observed relationship or rules in learner's own words <p>Input and output values</p> <ul style="list-style-type: none"> Determine the input and output values given rules in <ul style="list-style-type: none"> – flow diagram – tables Determine the rule given input and output values in flow diagrams <p>Equivalent forms</p> <ul style="list-style-type: none"> Determine equivalence of different descriptions of the <u>same relationship or rule</u> presented <ul style="list-style-type: none"> – verbally – in a flow diagram – by a number sentence 	<p>Investigate and extend geometric patterns</p> <ul style="list-style-type: none"> Describe observed rules of patterns involving constant difference in own words or by number sentence <p>Input and output values</p> <ul style="list-style-type: none"> Determine the rule given input and output values in flow diagrams and tables <p>Equivalent forms</p> <ul style="list-style-type: none"> Determine equivalence of different descriptions of the <u>same relationship or rule</u> presented <ul style="list-style-type: none"> – in a table
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TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
Number sentence	<p>Patterns with number bonds</p> <ul style="list-style-type: none"> • Addition and subtraction with number bonds for multiples of 10, 100, 1000 and 10 000 • Multiplication facts of whole numbers by 10, 100, and 1 000 • Division facts of whole numbers by 10, 100 and 1 000 resulting to whole numbers <p>Solving problems</p> <ul style="list-style-type: none"> • Solve and complete number sentence by inspection using the following <ul style="list-style-type: none"> – Properties of whole numbers <ul style="list-style-type: none"> ✓ Commutative, associative and distributive property ✓ Identity element of 0 ✓ Identity element of 1 – inverse operations <ul style="list-style-type: none"> ✓ Addition and subtraction ✓ Multiplication and division • Solve and complete number sentence by <ul style="list-style-type: none"> – trial and improvement – substitution <p>Describing a problem situation</p> <ul style="list-style-type: none"> • Write number sentence to describe problem situation 	<p>Patterns with number bonds</p> <ul style="list-style-type: none"> • Addition and subtraction with number bonds for multiples of 10, 100 and 1000 <p>Solving problems</p> <ul style="list-style-type: none"> • Solve and complete number sentence by inspection using the following <ul style="list-style-type: none"> – Properties of whole numbers <ul style="list-style-type: none"> ✓ Commutative, associative and distributive property – inverse operations <ul style="list-style-type: none"> ✓ Add Addition and subtraction ✓ Multiplication and division • Solve and complete number sentence by <ul style="list-style-type: none"> – trial and improvement – substitution <p>Describing a problem situation</p> <p>Write number sentence to describe problem situation</p>	<p>Patterns with number bonds</p> <ul style="list-style-type: none"> • Addition and subtraction with number bonds for multiples of 10 000 • Multiplication facts of whole numbers by 10, 100, and 1 000 • Division facts of whole numbers by 10, 100 and 1 000 resulting to whole numbers <p>Solving problems</p> <ul style="list-style-type: none"> • Solve and complete number sentence by inspection using the following <ul style="list-style-type: none"> – Properties of whole numbers <ul style="list-style-type: none"> ✓ Identity element of 0 ✓ Identity element of 1

SPACE AND SHAPE			
TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
Properties of 2-D_shapes	<p>Range of shapes</p> <ul style="list-style-type: none"> Recognize, visualize and name 2-D shapes in the environment and geometric settings <ul style="list-style-type: none"> regular and irregular polygons – right triangles, triangles, squares, rectangles, other quadrilaterals, pentagons, hexagons circles <p>Characteristics of shapes</p> <ul style="list-style-type: none"> Describe, sort and compare 2-D shapes in terms of <ul style="list-style-type: none"> straight and curved sides number of sides lengths of sides angles in shapes, limited to right angles <p>Further activities</p> <ul style="list-style-type: none"> Draw squares, rectangles and right triangles on a grid paper 	<p>Range of shapes</p> <ul style="list-style-type: none"> Recognize, visualize and name 2-D shapes in the environment and geometric settings <ul style="list-style-type: none"> regular and irregular polygons, triangles, squares, rectangles, other quadrilaterals, pentagons, hexagons circles <p>Characteristics of shapes</p> <ul style="list-style-type: none"> Describe, sort and compare 2-D shapes in terms of <ul style="list-style-type: none"> straight and curved sides number of sides <p>Further activities</p> <p>Draw 2-D shapes on a grid paper</p>	<p>Range of shapes</p> <ul style="list-style-type: none"> Recognize, visualize and name 2-D shapes in the environment and geometric settings <ul style="list-style-type: none"> regular and irregular polygons – right triangles, <p>Characteristics of shapes</p> <ul style="list-style-type: none"> Describe, sort and compare 2-D shapes in terms of <ul style="list-style-type: none"> lengths of sides angles in shapes, limited to right angles <p>Further activities</p> <p>Draw squares, rectangles and right triangles on a grid paper</p>
Properties of 3-D_objects	<p>Range of objects</p> <ul style="list-style-type: none"> Recognize, visualize and name 3-D objects in the environment and geometric settings, focusing on: <ul style="list-style-type: none"> rectangular prisms, spheres cylinders pyramids <p>characteristics of objects</p> <ul style="list-style-type: none"> Describe, sort and compare 3-D objects in terms of <ul style="list-style-type: none"> shapes of faces flat and curved surfaces <p>Further activities</p> <ul style="list-style-type: none"> Make 3-D models using cut out polygons 	<p>Range of objects</p> <ul style="list-style-type: none"> Recognize, visualize and name 3-D objects in the environment and geometric settings, focusing on: <ul style="list-style-type: none"> rectangular prisms, spheres cylinders pyramids <p>characteristics of objects</p> <ul style="list-style-type: none"> Describe, sort and compare 3-D objects in terms of <ul style="list-style-type: none"> shapes of faces flat and curved surfaces <p>Further activities</p> <p>Make 3-D models using cut out polygons</p>	N/A
Symmetry	<p>Symmetry</p> <ul style="list-style-type: none"> Recognize, draw and describe line(s) of symmetry in 2-D shapes 	<p>Symmetry</p> <p>Recognize, draw and describe line(s) of symmetry in 2-D shapes</p>	Done as early as FP

TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
Transformation	<p>Build composite shapes</p> <ul style="list-style-type: none"> Put 2-D shapes together to make different composite 2-D shapes including some shapes with line symmetry. <p>Tessellations</p> <ul style="list-style-type: none"> Pack out 2-D shapes to make tessellated patterns including some patterns with line symmetry. <p>Describe patterns (Revision)</p> <ul style="list-style-type: none"> Refer to lines, 2-D shapes, 3-D objects and lines of symmetry when describing patterns <ul style="list-style-type: none"> in nature from modern everyday life our cultural heritage 	<p>Build composite shapes</p> <ul style="list-style-type: none"> Put 2-D shapes together to make different composite 2-D shapes including some shapes with line symmetry. <p>Tessellations</p> <ul style="list-style-type: none"> Pack out 2-D shapes to make tessellated patterns including some patterns with line symmetry. <p>Describe patterns</p> <ul style="list-style-type: none"> Refer to lines, 2-D shapes, 3-D objects and lines of symmetry when describing patterns <ul style="list-style-type: none"> in nature from modern everyday life our cultural heritage 	<p>N/A</p> <p>N/A</p> <p>N/A, but this is done as early as FP</p>
Viewing objects	<p>Position and views</p> <ul style="list-style-type: none"> Match different views of everyday objects Identify everyday objects from different views 	<p>Position and views</p> <ul style="list-style-type: none"> Match different views of everyday objects Identify everyday objects from different views 	N/A
Position and movement	<p>Location and directions</p> <ul style="list-style-type: none"> Locate position of objects, drawings or symbols on a grid with alpha-numeric grid references Locate positions of objects on a map by using alpha-numeric grid references 	<p>Location and directions</p> <ul style="list-style-type: none"> Locate position of objects, drawings or symbols on a grid with alpha-numeric grid references Locate positions of objects on a map by using alpha-numeric grid references 	N/A

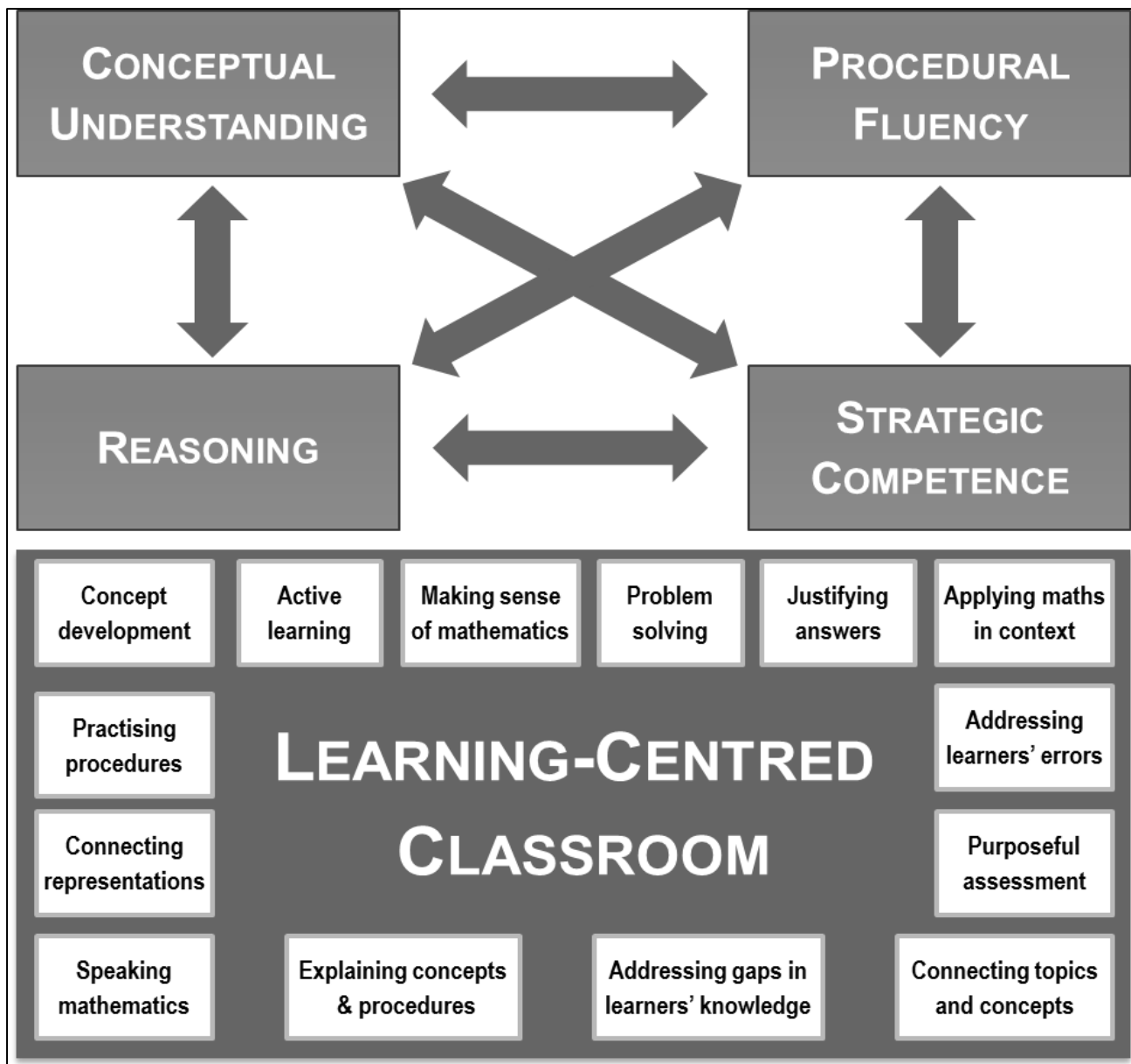
MEASUREMENT			
TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
Length	<p>Measurement, instruments and units</p> <ul style="list-style-type: none"> Estimate the length of 2-D shapes and 3-D objects Practically measure the length of 2-D shapes and 3-D objects using measuring instruments such as rulers, metre sticks, tape measures, trundle wheels Record the length of the above in units of length: millimetres (<i>mm</i>), centimetres (<i>cm</i>), metres (<i>m</i>), kilometres (<i>km</i>) <p>Calculations and problem-solving involving length</p> <ul style="list-style-type: none"> Solve problems in contexts related to length involving conversions include converting between: <ul style="list-style-type: none"> <i>mm</i> and <i>cm</i> <i>cm</i> and <i>m</i> <i>m</i> and <i>km</i> <p>N.B Conversions are limited to whole numbers, fractions and decimal fractions to one decimal place</p>	<p>Measurement, instruments and units</p> <ul style="list-style-type: none"> Estimate the length of 2-D shapes and 3-D objects Practically measure the length of 2-D shapes and 3-D objects using measuring instruments such as rulers, metre sticks, tape measures, trundle wheels Record the length of the above in units of length: millimetres (<i>mm</i>), centimetres (<i>cm</i>), metres (<i>m</i>), kilometres (<i>km</i>) <p>Calculations and problem-solving involving length</p> <ul style="list-style-type: none"> Solve problems in contexts related to length involving conversions include converting between: <ul style="list-style-type: none"> <i>mm</i> and <i>cm</i> <i>cm</i> and <i>m</i> <i>m</i> and <i>km</i> <p>N.B Conversions are limited to whole numbers and common fractions</p>	<p>Measurement, instruments and units N/A</p> <p>Calculations and problem-solving involving length</p> <p>N.B Conversions are limited to whole numbers, fractions and decimal fractions to one decimal place</p>
Mass	<p>Measurement, instruments and units</p> <ul style="list-style-type: none"> Estimate the mass of 3-D objects Practically measure the mass of 3-D objects using measuring instruments such as bathroom scales, kitchen scales and balances Record the mass of the above in units of mass: grams (g) and kilograms (kg); <p>Calculations and problem-solving involving mass include</p> <ul style="list-style-type: none"> problems in contexts converting between grams and kilograms limited to examples with whole numbers, fractions and decimal fractions to one decimal place 	<p>Measurement, instruments and units</p> <ul style="list-style-type: none"> Estimate the mass of 3-D objects Practically measure the mass of 3-D objects using measuring instruments such as bathroom scales, kitchen scales and balances Record the mass of the above in units of mass: grams (g) and kilograms (kg); <p>Calculations and problem-solving involving mass include</p> <ul style="list-style-type: none"> problems in contexts converting between grams and kilograms limited to examples with whole numbers and common fractions 	<p>Measurement, instruments and units N/A</p> <p>Calculations and problem-solving involving mass include</p> <ul style="list-style-type: none"> converting between grams and kilograms limited to examples with whole numbers, fractions and decimal fractions to one decimal place

TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
Capacity/ volume	<p>Measurement, instruments and units</p> <ul style="list-style-type: none"> Estimate the capacity/volume of 3-D objects Practically measure the capacity/volume of objects using measuring instruments such as measuring spoons, measuring cups, measuring jugs Record the capacity/volume of the above in units of mass: millilitres (<i>ml</i>) and litres (<i>l</i>) <p>Calculations and problem-solving involving capacity/volume include</p> <ul style="list-style-type: none"> problems in contexts converting between millilitres (<i>ml</i>) and litres (<i>l</i>) limited to examples with whole numbers, fractions and decimal fractions to one decimal place 	<p>Measurement, instruments and units</p> <ul style="list-style-type: none"> Estimate the capacity/volume of 3-D objects Practically measure the capacity/volume of objects using measuring instruments such as measuring spoons, measuring cups, measuring jugs Record the capacity/volume of the above in units of mass: millilitres (<i>ml</i>) and litres (<i>l</i>) <p>Calculations and problem-solving involving capacity/volume include</p> <ul style="list-style-type: none"> problems in contexts converting between millilitres (<i>ml</i>) and litres (<i>l</i>) limited to examples with whole numbers and common fractions 	<p>Measurement, instruments and units</p> <ul style="list-style-type: none"> N/A <p>Calculations and problem-solving involving capacity/volume include</p> <ul style="list-style-type: none"> converting between millilitres (<i>ml</i>) and litres (<i>l</i>) limited to examples with whole numbers, fractions and decimal fractions to one decimal place
Area and Perimeter and volume of solids	<p>Perimeter Calculate the perimeter of regular and irregular shapes by adding the lengths of the sides.</p> <p>Measurement of area Calculate the area of squares and rectangles</p> <p>Measurement of volume Calculate the volume of simple solid objects</p>	<p>Perimeter Measure perimeter using rulers or measuring tapes</p> <p>Measurement of area</p> <ul style="list-style-type: none"> Find areas of regular and irregular shapes by counting squares on grids in order to develop an understanding of square units <p>Measurement of volume</p> <ul style="list-style-type: none"> Find volume/capacity of objects by packing or filling them in order to develop an understanding of cubic units 	<p>Perimeter N/A</p> <p>Measurement of area Calculate the area of squares and rectangles</p> <p>Measurement of volume Calculate the volume of simple solid objects</p>
Time	<p>Reading time and time instruments</p> <ul style="list-style-type: none"> Read, tell and write time in 12-hour and 24-hour formats on both analogue and digital instruments in <ul style="list-style-type: none"> hours minutes seconds Instruments include clocks and watches <p>Reading calendars</p> <p>Calculations and problem solving time include</p> <ul style="list-style-type: none"> problems in contexts involving time calculation of the number of days between any two dates within the same or consecutive years calculation of time intervals where time is given in minutes or hours only <p>History of time Know some ways in which time was measured and represented in the past</p>	<p>Reading time and time instruments</p> <ul style="list-style-type: none"> Read, tell and write time in 12-hour and 24-hour formats on both analogue and digital instruments in <ul style="list-style-type: none"> hours minutes seconds Instruments include clocks and watches <p>Reading calendars</p> <p>Calculations and problem solving time include</p> <ul style="list-style-type: none"> problems in contexts involving time calculation of the number of days between any two dates within the same or consecutive years calculation of time intervals where time is given in minutes or hours only <p>History of time Know some ways in which time was measured and represented in the past</p>	N/A

DATA HANDLING

TOPIC	CONCEPTS AND SKILLS	CAPS	NEW/ EXTENSION
5.1 Collecting and Organising data	Collecting and organising data <ul style="list-style-type: none"> Collect data using tally marks and tables for recording 	Collecting and organising data <ul style="list-style-type: none"> Collect data using tally marks and tables for recording 	N/A
5.2 Representing data	Representing data <ul style="list-style-type: none"> Draw a variety of graphs to display and interpret data including: <ul style="list-style-type: none"> pictographs (many-to-one correspondence) between data and representation) bar graphs 	Representing data <ul style="list-style-type: none"> Draw a variety of graphs to display and interpret data including: <ul style="list-style-type: none"> pictographs (one-to-one correspondence between data and representation) bar graphs 	Representing data <ul style="list-style-type: none"> Draw a variety of graphs to display and interpret data including: <ul style="list-style-type: none"> pictographs (many-to-one correspondence) between data and representation)
5.3 Analysing, Interpreting and Reporting data	Interpreting data <ul style="list-style-type: none"> Critically read and interpret data represented in <ul style="list-style-type: none"> words pictographs bar graphs pie charts Analysing data Analyse data by answering questions related to data categories	Interpreting data <ul style="list-style-type: none"> Critically read and interpret data represented in <ul style="list-style-type: none"> words pictographs bar graphs pie charts Analysing data Analyse data by answering questions related to data categories	N/A
5.4 Probability	N/A	Probability experiments <ul style="list-style-type: none"> Perform simple repeated events and list possible outcomes for experiments such as: <ul style="list-style-type: none"> tossing a coin rolling a die 	Delay in teaching probability

Mathematics Teaching and Learning Framework



**A: TEACHING MATHS FOR CONCEPTUAL UNDERSTANDING
(Maths Teaching and Learning Framework p. 15)**

Conceptual knowledge is knowledge of concepts, relations, and patterns. It assists and enables learners to make sense of mathematics. Learners who have a sound grasp of conceptual knowledge, when asked to justify their work, would say, 'My teacher told me to do it like this' rather they will be able to explain the reasoning behind their work.

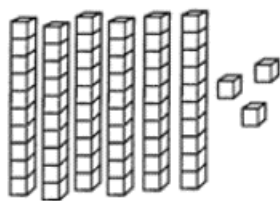
Learners who have conceptual knowledge are able to compare, relate and infer. They can make connections between ideas. Higher level thinking is fundamental to conceptual knowledge. Conceptual knowledge is constructed as problems are solved, investigations are carried out, and questions are pondered. Meanings and connections among ideas develop as learners work with concrete, pictorial, and symbolic material, as they reflect on what they have done, and as they communicate with others. However, learners often need teachers to thoughtfully and strategically push them to progress from concrete, specific working to more abstract generalised ideas. The more the learners are exposed to such ways of working, the better the chances become that they will develop into mathematical problem solvers.

When a new concept is introduced to learners, the teacher should plan a lesson for conceptual development. This process places considerable pedagogic demands on the teacher. All introductory lessons should be carefully planned. Learners could be engaged by sharing their ways of thinking and the teacher should use these ideas to challenge learners to construct and connect ideas through reflective discussions on the methods.

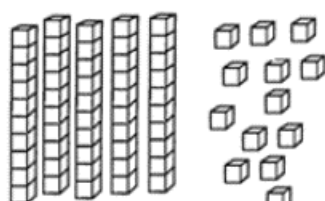
Teachers might choose to use appropriate manipulatives and bring in aspects of learners' out of school experiences in order to help learners to build their knowledge.

For example, a learner finding the difference between two 2-digit numbers could be shown how the calculation could be done using base ten blocks. The concept of subtraction, which was introduced in Grade 1 is reinforced in this Grade 2 example, by the concrete demonstration, which can be linked to a numeric calculation to help the learner make a connection between concrete working and abstract calculations.

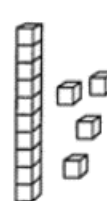
$63 - 49 =$



Layout 63 using base ten blocks



Exchange one ten for ten ones



14 remains after subtracting 49

I have 6 tens and 3 ones from which I must subtract 49.

I have 5 tens and 13 ones after I exchange in order to be able to subtract.

If I take away 4 tens and 9 ones, I have 1 ten and 4 ones left, that is 14 left.

The concrete working with base ten blocks shown above demonstrates the exchange needed in order to do subtraction. This concrete activity builds up learners' conceptual understanding of number and operations. The written record of the procedure is based on conceptual knowledge. Learners need connections to be made explicit in order to become fluent in the use of procedures.

1. What is conceptual knowledge? Write a short definition / description.

Conceptual knowledge is knowledge of concepts, relations and patterns. Conceptual knowledge assists and enables learners to make sense of maths. Learners who have conceptual knowledge are able to explain the reasoning behind their work.

2. What are learners who have conceptual knowledge able to do?

Learners with conceptual knowledge are able to compare, relate (show connections between things) and infer (conclude from the given information)

3. How is concept knowledge constructed?

Conceptual knowledge is constructed as problems are solved, investigations are carried out and questions are pondered (thought about).

4. How are meanings and connections among ideas develop?

Meanings and connections among ideas develop as learners work with the concrete (actual Base 10 blocks), the pictorial (pictures of Base 10 blocks) and the symbolic (written calculations); as they reflect on what they have done; as they communicate with others. The more the learners are exposed to such ways of working, the better the chances become that they will develop into mathematical problem solvers.

5. What should teachers plan for when introducing a new concept to learners?

When introducing a new concept to the learners, the teacher should always plan an introductory lesson which allows for conceptual development. Teachers should use the learners' ways of thinking to encourage the learners to construct and connect ideas through reflective discussions (where learners are encouraged to think and talk about the methods).

6. In the example given where the learners have to subtract 49 from 63, what conceptual knowledge is being developed?

Learners in the higher grades often struggle when having to "borrow" when subtracting. Allowing the learners to develop an understanding that larger numbers are being exchanged or broken up in order to assist with the subtraction will assist the learners conceptual understanding of subtracting with "borrowing".

B: TEACHING MATHS FOR PROCEDURAL FLUENCY (Maths Teaching and Learning Framework p. 16)

Whereas conceptual understanding is an implicit or explicit understanding of the interrelations between pieces of knowledge, procedural knowledge is seen as the sequence of actions that are performed to solve a problem. The two types of knowledge do not develop independently. Conceptual understanding often leads into the procedures that the learner will use. Often conceptual understanding precedes procedural skills. A teacher who is aware of the importance of conceptual understanding when teaching concepts will not teach the procedural skills before learners have mastered the concepts involved.

Procedural knowledge is the recognition of symbols and the ability to follow rules to 'do' mathematics. It can be thought of as having mathematical skills and carrying out actions in a correct sequence. Mathematical expertise involves both conceptual and procedural knowledge and also awareness that procedures are based on mathematical principles. Conceptual and procedural knowledge support each other and work together to attain mathematical power. Procedures connected to conceptual knowledge give flexibility to mathematical thinking and enable learners to extend the range of both types of knowledge when new problems arise. If children learn procedures without understanding, their knowledge may be limited to meaningless routines.

Research has shown that increasing learners' conceptual knowledge leads to the ability to generate one's own procedures. There is a reciprocal relationship between conceptual and procedural knowledge, but as argued above, conceptual knowledge has a stronger and more foundational role to play in developing procedural knowledge than the reverse. Conceptual understanding leads to the generation of flexible procedures, and procedural knowledge can lead to conceptual understanding. Similarly, conceptual teaching (teaching focused on developing conceptual understanding) enables the effective teaching of flexible procedures and these procedures then enable strengthened engagement with conceptual understanding in one's teaching.

Developing procedural fluencies for adding and subtracting numbers is essential for further mathematical learning. Fluency is developed through much repetition and practice.

For example, a teacher could teach learners how to use the vertical algorithm when teaching addition and subtraction (the additive relations). The vertical method of recording working with numbers links to the number system that we use and can be explained using place value. It is an efficient and effective procedure that has been used for many years. Correct working with vertical algorithms builds on and develops learners' number sense but only if learners understand why the 6 becomes 5 and the 3 becomes 13 in the steps used.

$$\begin{array}{r} 63 - 49 = \square \\ \begin{array}{r} 5 \quad 13 \\ \cancel{6} \quad \cancel{3} \\ - \quad 4 \quad 9 \\ \hline 1 \quad 4 \end{array} \end{array}$$

1. What is a procedure in maths? Give an example of a procedure.

A procedure is a way of doing something. An algorithm is a procedure. The addition algorithm is the procedure that we use when adding numbers vertically.

2. What is procedural knowledge? Write a short definition / description.

Procedural knowledge is the recognition of symbols and the ability to follow rules to 'do' mathematics. It can be thought of as having mathematical skills and carrying out actions in a correct sequence.

3. Generally, which comes first: conceptual understanding or procedural fluency?

These two types of knowledge do not develop independently. Conceptual understanding often leads into the procedures that a learner will use. Often conceptual understanding precedes procedural skills. A teacher who is aware of the importance of conceptual understanding when teaching concepts will not teach procedural skills before learners have mastered the concepts involved.

4. What has research shown about the relationship between conceptual and procedural knowledge?

Conceptual knowledge is constructed as problems are solved, investigations are carried out and questions are pondered (thought about).

5. How are meanings and connections among ideas develop?

Research has shown that increasing a learner's conceptual knowledge leads to the ability to generate one's own procedures.

There is a reciprocal relationship between conceptual and procedural knowledge, conceptual knowledge makes a greater contribution to procedural knowledge, than vice versa.

6. How is procedural fluency developed in the learners?

Fluency is developed through much repetition and practice.

7. Assume that learners have already been introduced to the concept of "borrowing" when subtracting using Base 10 blocks. (See the example in A.) Explain in detail how you would explain the procedure being used in the vertical subtraction of $63 - 49$.

$63 = 6 T + 3 O$. $49 = 4 T + 9 O$.

We cannot subtract $9 O$ from $3 O$.

We go to $6 T$ and exchange it for $5 T$ and $10 O$.

This means that we now have $10 O + 3 O = 13 O$.

We can now subtract $9 O$ from $13 O$ and get $4 O$.

And we can subtract $4 T$ from $5 T$ and get $1 T$.

So, the answer is $1 T + 4 O$ or 14 .

C: DEVELOPING LEARNERS' STRATEGIC COMPETENCE / OWN STRATEGIES
(Maths Teaching and Learning Framework p. 17)

Modern societies and economies are in a constant state of flux. It is no longer sufficient for learners only to learn how to reproduce the steps in the calculations that they are shown by teachers. If we are to equip learners for the future, they need to be adaptable: to use knowledge and skills in flexible ways; to be able to retrieve knowledge and apply skills as the basis for new learning. Mathematics teaching and learning should develop learners who are able to adapt their knowledge and use their skills flexibly, not only as the basis for solving problems, but also as the foundation for learning new skills and knowledge.

Learners should be able to make sensible decisions on what strategies to employ or to devise their own strategy to solve certain problems. Often there is more than one way to solve a mathematical problem and it is important that learners do not always depend on fixed, prescribed methods to solve problems. This dimension includes two (related) skills:

- a) *Strategic competence* - the ability to formulate, represent and solve mathematical problems. Learners should be able to read and make sense of a mathematical problem, look for possible patterns and use some strategy to solve the problem. There may be a variety of strategies that are useful in different contexts. Part of strategic competence is the ability to select and use an appropriate strategy in a given context.
- b) Learners using their *own strategies* to approach a problem that cannot be solved using familiar strategies. Learners tend to focus on technical, procedural aspects of mathematics and learners do not get the much needed exposure to *problem solving* which is an integral part of mathematics. Learners need this exposure in order to develop the capacity to generate their own strategies since this is a basis for problem solving. Learners need to have and be able to use procedures which they have at their disposal.

It is particularly useful to allow learners to use their own strategies when introducing a topic or a new section of a topic, because it can help them to develop a better conceptual understanding of the topic. When learners discuss and compare their own strategies they can learn ways of reasoning and improving their calculations from their peers. When learners try out new strategies they can find out whether the initial strategy used is particular to the example, or whether it is generalisable to many examples.

For example, learners could use many different strategies to calculate subtraction. Two examples of strategies are shown below. There are many other possibilities! The strategies below should not be taught in a rote fashion as this could lead to misconceptions or incorrect use of the procedure. Correct working with mathematical strategies builds on and develops learners' number sense but only if learners understand how the procedure works.

$64 - 49 = \square$

<p>Strategy 1 $63 - 49 = 63 - 40 - 9$... the learner broke down 49 into 40 and 9</p> $= 23 - 9$ $= 14$	<p>Strategy 2 $63 - 49 = 63 - 50$ then add 1 back</p> $= 13 + 1$ $= 14$ <p>This can be shown on a number line</p>
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Ultimately, learners should be guided to choose which strategy they find most efficient and prefer. They should also learn how to choose which strategy is most appropriate in a given context.

Learners need to develop their ability to *think out of the box* (i.e. to find strategies that have not been shown to them before) since this is useful for effective problem solving in mathematics. Bearing this in mind, the framework provides an example per phase of a more 'open' problem in addition to the more standard curriculum linked examples. When learners 'think out of the box' they are applying mathematical reasoning.

The beginning of 'out of the box' thinking is when learners are able to act independently – moving towards

fluency and efficiency in the use of procedures in the solution of problems. An effective way of giving learners exposure to problem solving is through participation in mathematics competitions. These competitions include questions that learners have not generally seen before and for which standard taught procedures will not be sufficient to solve the problems. The perception that 'mathematics competitions are only for the gifted learner' is a myth. All learners, not only gifted learners, need to develop the thinking skills needed to solve non-standard problems. There are two official national events, the SA Mathematics Challenge (SAMC) for primary school learners, and the SA Mathematics Olympiad (SAMO) for secondary school learners (both organised by the SA Mathematics Foundation and endorsed by the national Department of Basic Education)³. There are also other regional or local mathematics competitions. Participating in these competitions could enable learners to grow their mathematical thinking skills.

1. What is strategic competence? Write a short definition / description.

Strategic competence is the ability to formulate, represent and solve mathematical problems. Learners should be able to read and make sense of a mathematical problem, look for possible patterns and use some strategy to solve the problem. Part of strategic competence is the ability to select and use an appropriate strategy in a given context.

2. Why do learners need exposure to problem solving?

They need exposure to problem solving in order to develop their capacity to generate their own strategies since this is a basis for problem solving.

3. How can using their own strategies help when introducing a new topic?

Using their own strategies helps the learner to develop a better conceptual understanding of the topic. When learners discuss and compare their own strategies, they can learn ways of reasoning and improving their calculations from their peers. As they try out new strategies, they can find out whether the initial strategy used is particular to that example or whether it can be generalised to many examples.

4. What do we mean by "thinking out of the box"?

Thinking out of the box means finding strategies that have not been shown to them before. Thinking out of the box is useful for effective problem solving in maths. When learners think out of the box, they are applying mathematical reasoning.

5. How can we help learners develop "out of the box" thinking?

We can get the learners to practice answering questions from maths competitions and then getting the learners to enter maths competitions. Taking part in these competitions enables the learners to grow their mathematical skills.

D: DEVELOPING LEARNERS MATHEMATICAL REASONING (Maths Teaching and Learning Framework p. 15)

Many learners see mathematics as a system of algorithms to be performed to get 'the right' answer. Along with other components such as creativity and intuition, logic forms an integral part of mathematical thinking, however even adults sometimes find it difficult to reason in a formal logical way. To help learners reason mathematically, we need to teach them skills they do not possess naturally.

Mathematics is not simply a collection of isolated procedures and facts; it consists of a web of interconnected concepts and relationships. If learners are taught mathematics as a series of disconnected procedures that need to be learnt by heart, they are likely to experience mathematics as meaningless. It will also mean that they have more to memorise which deprives them of the opportunity to develop higher order thinking skills. If, on the other hand, learners are encouraged to connect topics and develop the practice of thinking '*What do I already know, that can help me here*' they will reduce the strain on their memory and increase their reasoning abilities.

Learners use *inductive reasoning* to make generalisations based on evidence they have found. In this kind of reasoning facts are usually accumulated to convince us of the validity of a particular statement. *Inductive reasoning* starts with specific examples or observations and leads to a conjecture about the apparent rules or patterns that lie behind them. A mathematical example of *inductive reasoning* is the identification of a pattern in a sequence of numbers.

A large part of mathematics is based on an axiomatic system in which *deductive reasoning* is the accepted route to gain new mathematical knowledge. *Deductive reasoning* starts with the rules (or axioms), and through *deductive reasoning* we determine what the consequences will be. This is what mathematicians do in most of mathematics, defining the rules for a mathematical entity (such as the basic axioms for Euclidean Geometry), and using these rules to prove that other more complicated facts are true. With *deductive reasoning* we can be absolutely sure of our conclusions - as long as we assume the axioms are true.

Logical deductive reasoning is an important foundational skill in mathematics. Learning mathematics is a sequential process of building connections. If learners do not fully grasp a certain concept or procedure, they may struggle to understand other concepts or procedures that follow – since these may depend on or build on the earlier ideas. Disciplined deductive mathematical reasoning is crucial to understanding and to using mathematics properly.

Reasoning mathematically involves learners talking about mathematics. Learners must learn to speak the language of mathematics for themselves. They cannot do this without being given opportunities to 'talk mathematics'. Teachers should support learners as they learn to develop their mathematical language. At times learners may be able to give answers but struggle to explain how they got to the answers. Teachers should support learners to develop the language and skills needed to talk about their thinking, answers and solution strategies.

As learners progress they need to learn to work (and speak about mathematical objects) more abstractly. This requires them to start to reason by making use of formal mathematical definitions in order to justify their answer or to build an argument.

For example, learners need to move from a simplistic claim that a shape is a rectangle: 'it is a rectangle because it is a longish shape with two long sides and two short sides and four corners' to a more sophisticated argument: 'a shape is a rectangle if its opposite sides are equal **and** it has four right angles'. In the first case learners will not acknowledge that a square is also a rectangle while in the more formal abstract discourse a square is clearly a special case of a rectangle.

In the example below the Grade 2 learner's answer shows the use of mathematical language to explain how they reasoned when finding a solution based on given information.

Question: If $63 - 49 = 14$, then what is $63 - 39$?

A learner used reasoning to answer as follows:

I am subtracting 10 less and so the answer must be 10 more.
The answer is 24.

1. What combines to form mathematical reasoning.

Creativity, intuition (the ability to understand something without the need for reasoning), and logic form an integral part of mathematical reasoning.

2. Is maths just a collection of isolated procedures and facts?

No, it isn't. it consists of a mass of interconnected concepts and relationships.

3. What happens if learners are taught maths as a series of disconnected procedures that have to be learned off by heart?

Learners will experience maths as meaningless. They will spend time memorising facts and don't have time to develop higher order thinking skills. They should be encouraged to ask themselves "what do I already know that can help me here?"

4. What is meant by Inductive Reasoning? Give an example of Inductive Reasoning.

Inductive reasoning starts with specific examples or observations and leads to a conjecture (an opinion or conclusion) about the apparent rules or patterns that lie behind these examples or observations.

An example of inductive reasoning is identifying a pattern in a sequence of numbers.

5. What is meant by Deductive Reasoning? Give an example of Deductive Reasoning.

Deductive reasoning starts with rules (or axioms). We use deductive reasoning to determine what the consequences will be.

An example of deductive reasoning is using axioms and theorems in Euclidian Geometry to solve geometry problems or riders.

6. Why should learners be encouraged to talk about maths?

Reasoning mathematically involved learners talking about maths. Teachers should support learners as they learn to develop their mathematical language and skills needed to talk about their thinking, their answers and their solution strategies.

E: PROMOTING AND NURTURING A LEARNING CENTRED CLASSROOM (Maths Teaching and Learning Framework p. 19)

A learning-centred classroom creates a platform for meaningful learning and teaching. The framework diagram illustrates this by placing the learning-centred classroom as the foundation for all of the other dimensions of the framework. Teachers need to create classrooms where the stage is set for learning mathematics for understanding. The term "learning-centred" has been chosen very deliberately. Much has been written and said about *learner-centred* and *teacher-centred* classrooms. Often it has been suggested that "teacher-centred is bad" and "learner-centred is good". Such dichotomies are not helpful and they are not accurate. There are many reports of so-called learner-centred classrooms where it is not clear what learners were supposed to be learning. On the other hand, we read of so-called teacher-centred classrooms where learners displayed a good grasp of what their teacher had explained using a so-called chalk-and-talk approach.

A **learning**-centred classroom focuses on *learning* – where the teacher designs learning experiences to help learners learn mathematics, using whatever teaching and learning strategies s/he thinks are most suitable for the specific lesson that will be taught.

A **learning**-centred mathematics classroom is characterised by a culture of interaction between teachers and learners in the process of 'doing mathematics'. The teacher plays an important role in establishing and nurturing this culture. The way in which a teacher conducts a classroom, depends on the way in which s/he views mathematics. A teacher who sees mathematics as a body of knowledge which s/he has to impart to learners, will mostly tell learners what to do, and how to do it. On the other hand, a teacher who sees mathematics as a body of knowledge that learners must actively explore and engage with, will create a learning environment where learners can make sense of mathematics. They will have opportunity to express their ideas, to ask questions of the teacher and each other and discuss their ways of thinking about the work at hand.

Teachers must direct and be in control of the path of learning but they must see the learners' role as active in developing understanding and taking ownership of what they have learned. For example, in the Foundation Phase, focused and guided play-based learning should be promoted because it is an important component of active learning of mathematics.

The CAPS Grades R - 12 aims to develop citizens that are able to:

- identify and solve problems and make decisions using critical and creative thinking;
- work effectively as individuals and with others as members of a team;
- organise and manage themselves and their activities responsibly and effectively;
- collect, analyse, organise and critically evaluate information;
- communicate effectively using visual, symbolic and/or language skills in various modes;
- use science and technology effectively and critically showing responsibility towards the environment and health of others; and
- demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

It is evident that the CAPS emphasises the importance of developing learners' ability to solve problems and to share and communicate their ideas. In a **learning**-centred mathematics classroom learners will have opportunity to:

- make sense of mathematics
- speak mathematics
- develop fluency in essential mathematical procedures
- connect representations
- justify their thinking

In a **learning**-centred mathematics classroom teachers will:

- use assessment for learning
- provide clear explanations of concepts and procedures
- address learners' errors and misconceptions
- address gaps in learners' knowledge
- make connections between different topics
- provide opportunities for active learning
- select and design tasks that emphasise key mathematical ideas and ways of working mathematically
- encourage learners to speak mathematics and use mathematical notation accurately

All four dimensions of the framework are closely linked and in a learning-centred classroom they will interact dynamically. Some dimensions will come to the foreground in some lessons while other dimensions will come more strongly to focus in other lessons. All this comes together as teachers strive to teach mathematics for understanding to enable learners to learn mathematics with understanding.

An interactive lesson can develop around examples which a teacher has written on the board or in the context of an activity planned by the teacher.

Teaching in an environment where learners are active and talk through their own learning calls for careful teacher preparation. Teachers might have to take on a different role, and that could make them feel insecure. A learning-centred classroom calls for more freedom on the part of learners. This requires careful planning from the teacher.

The teacher must prepare the lessons in much more detail, taking into consideration the possible questions and problems that learners could encounter. As teachers work consistently towards creating a learning-centred classroom environment, they will become more familiar with the type of questions and problems that learners encounter.

Teachers will be better prepared to teach interactively as they become more experienced in creating a learning-centred classroom. Once the initial work has been done to create scenarios where learners are involved in solving problems collaboratively and independently, the teaching load will become less.

Teachers are sometimes under the impression that they must control all of the learning that takes place in their classrooms when in fact this is not the case. Teachers should be in control of the learning pathways they direct, but they must do this to create independent learners.

Learners must take responsibility for and control of their own learning in order to become independent learners and thinkers, able to operate mathematically in the world without the teacher.

The role of the teacher should thus be to create an environment in which learners are provided rich activities through which they can develop their independence and control their own learning. To create such a classroom is not always easy but the benefits to be reaped are those mentioned above – independent learners, capable of thinking and working on their own and doing mathematics in a meaningful way.

1. Look at the framework diagram again and confirm that the diagram places the learning-centred classroom as the foundation for all of the other dimensions of the framework.
2. What is a learning-centred classroom?

A learning centred classroom focusses on learning where the teacher designs learning experiences to help learners learn mathematics.

3. What culture is found in a learning centred classroom?

There is a culture of interaction between teacher and learners, both involved in 'doing mathematics'.

4. What will learners in a leaning centred classroom have the opportunity to do?

They will have an opportunity to make sense of maths; speak maths; develop fluency in essential maths procedures; make connections; justify their thinking.

5. What is the role of the teachers in a learning-centred maths classroom?

The teachers will use assessment for learning; provide clear explanations of concepts and procedures; address learners' errors and misconceptions; address gaps in learners' knowledge; make connections between different topics; provide opportunities for active learning; select and design tasks that emphasise key mathematical ideas and ways of working mathematically; encourage learners to speak mathematics and use mathematical notation correctly.

6. Teaching in an environment where learners are active and talk through their learning calls for careful teacher preparation. What points should the teacher consider in order to create a learning centred classroom?

- ***The teacher must prepare the lessons in more detail, taking into consideration the possible questions and problems that learners could encounter. (As teachers work consistently towards creating a learning-centred classroom environment, they will become more familiar with the type of questions and problems that learners encounter.)***
- ***Teachers need to be prepared to teach interactively. Once the initial work has been done to create scenarios where learners are involved in solving problems collaboratively and independently, the teaching load will become less.***
- ***Teachers should not feel that they must control of all of the learning that takes place in their classrooms. Teachers should be in control of the learning pathways they direct, but they must do this to create independent learners.***
- ***The teacher needs to create an environment in which learners are provided rich activities through which they can develop their independence and control their own learning. Learners must take responsibility for and control of their own learning in order to become independent learners and thinkers, able to operate mathematically in the world without the teacher. To create such a classroom is not always easy but the benefits to be reaped are those mentioned above – independent learners, capable of thinking and working on their own and doing mathematics in a meaningful way.***

Using Simplified Base 10 blocks

In the first term, Grade 4 learners have to use the column method to

- add numbers up to 5 digits where the sum is either a 5-digit number or a 6-digit number
- subtract 6-digit numbers (borrowing once / borrowing twice / borrowing from hundreds to calculate Ones).

NOTE: The column method is also sometimes referred to as the vertical method or vertical addition and subtraction.

To assist the learners to understand the column method, it is strongly suggested that each step of a vertical addition and subtraction is scaffolded by the use of Base 10 Blocks.

Base 10 Blocks

- A Base 10 Kit containing photocopyable large blocks is given in the Grade 4 Teacher Resource.
- A smaller photocopyable Base 10 Kit is given in the Grade 4 Learner Activity Book for Term 1.
- Simplified pictures of the Base 10 blocks can be drawn on the board. (*This recommended for Grade 4*). This is known as a simplified pictorial representation.

1) USING BASE 10 BLOCKS TO HELP LEARNERS TO ADD LARGE NUMBERS VERTICALLY

Example 1: $1\ 564 + 2\ 999 = \square$

SOLUTION

Step 1: Set out the Base 10 blocks with the vertical addition next to it:

Th	H	T	O
		 1	
		 1	




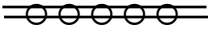

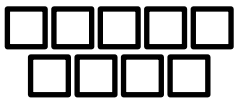


	Th	H	T	O
	1	5	6	4
+	2	9	9	9
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Step 2: Add the Ones. We get 13 Ones.

Th	H	T	O
		 1	
		 1	




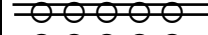

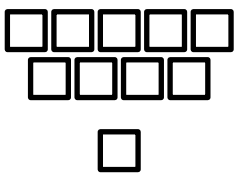


	Th	H	T	O
	1	5	6	4
+	2	9	9	9
<hr/>				

Step 3: Exchange 13 Ones for 1 ten and 3 Ones.

Th	H	T	O
			
			

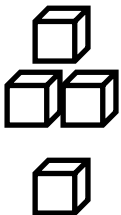
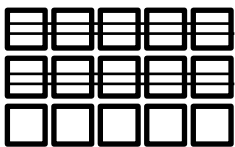

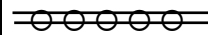


	Th	H	T	O
			1	
	1	5	6	4
+	2	9	9	9
				3

Step 4: Add the tens. We have 16 tens. Exchange 16 tens for 1 hundred and 6 tens.

Th	H	T	O
			
			

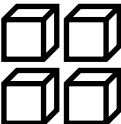
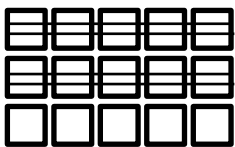

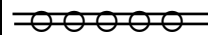


	Th	H	T	O
	1	1	1	
	1	5	6	4
+	2	9	9	9
			6	3

Step 5: We have 15 hundreds. Exchange 15 hundreds for 1 thousand and 5 hundreds.

Th	H	T	O
			
			

	Th	H	T	O
	1	1	1	
	1	5	6	4
+	2	9	9	9
		5	6	3

Step 6: We have 4 thousands.

Th	H	T	O
			
			

	Th	H	T	O
	1	1	1	
	1	5	6	4
+	2	9	9	9
	4	5	6	3

ANSWER: $1\,564 + 2\,999 = 4\,573$

I was then planning on giving the participants 2 different examples involving “carrying” and asking them to work in pairs describing what to do.

2) USING BASE 10 BLOCKS TO HELP LEARNERS TO SUBTRACT LARGE NUMBERS USING THE VERTICAL OR COLUMN METHOD

Example 1: $3\ 165 - 1\ 432 = \square$

SOLUTION

Step 1: Set out the Base 10 blocks showing the FIRST NUMBER ONLY with the vertical subtraction next to it.

Th	H	T	O

	Th	H	T	O
	3	1	6	5
-	1	4	3	2

Step 2: We start with the Ones. Subtract 2 Ones from 5 Ones.

Th	H	T	O
			We are left with 3 Ones

	Th	H	T	O
	3	1	6	5
-	1	4	3	2
				3


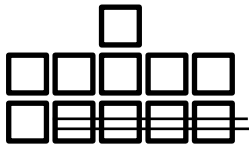
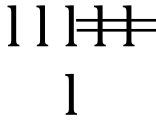
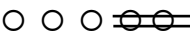
Step 3: Now subtract 3 tens from 6 tens.

Th	H	T	O
		We are left with 3 tens	We are left with 3 Ones

	Th	H	T	O
	3	1	6	5
-	1	4	3	2
			3	3


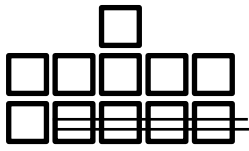
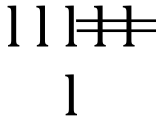

Step 4: We cannot subtract 4 hundreds from 1 hundred.

We borrow 1 thousand from the 3 thousands we have. We exchange 1 thousand for 10 hundreds. We add the 10 hundreds to the 1 hundred we already have. Now we have 11 hundreds.

Th	H	T	O
		 1	
We have 2 thousands left	Subtract 4 hundreds from the 11 hundreds We are left with 7 hundreds	We are left with 3 tens	We are left with 3 Ones

	Th	H	T	O
	2			
	3	11	6	5
-	1	4	3	2
		7	3	3

Step 5: Now we subtract 1 thousand from 2 thousands.

Th	H	T	O
		 1	
We are left with 1 thousands	Subtract 4 hundreds from the 11 hundreds We are left with 7 hundreds	We are left with 3 tens	We are left with 3 Ones

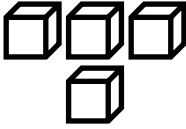

	Th	H	T	O
	2			
	3	11	6	5
-	1	4	3	2
	1	7	3	3

ANSWER: $3\ 165 - 1\ 432 = 1\ 733$

Example 2: $4\,500 - 1\,327 = \square$

SOLUTION

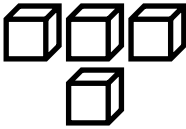
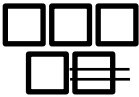
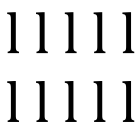
Step 1: Set out the Base 10 blocks showing the FIRST NUMBER ONLY with the vertical subtraction next to it.

Th	H	T	O
			

	Th	H	T	O
		4	5	0
-	1	3	2	7

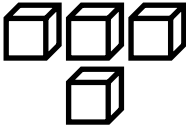

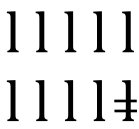
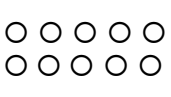
We cannot subtract 7 Ones from 0 Ones, so we go to the tens column.
 We cannot borrow from the tens column, so we go to the hundreds column where we have 5 hundreds.

Step 2: We borrow 1 hundred from the 5 hundreds we have. We exchange 1 hundred for 10 tens. We now have 4 hundreds and 10 tens

Th	H	T	O
			

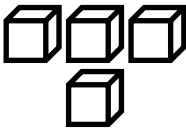
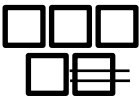
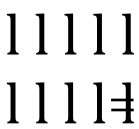
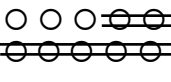
	Th	H	T	O
		4		
	4	5	10	0
-	1	3	2	7

Step 3: We cannot subtract 7 Ones from 0 Ones, so we borrow 1 ten from the tens column.
 We exchange 1 ten for 10 Ones. *We say we have borrowed 1 ten.*
 We now have 9 tens and 10 Ones.

Th	H	T	O
			

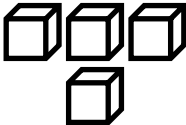
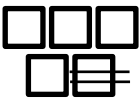
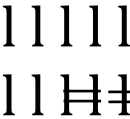
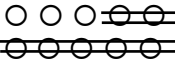
	Th	H	T	O
		4	9	
	4	5	10	10
-	1	3	2	7

Step 4: We can now subtract 7 Ones from the 10 Ones we have. We are left with 3 Ones.

Th	H	T	O
			

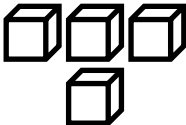

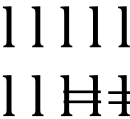
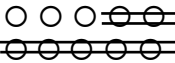
	Th	H	T	O
		4	9	
	4	5	10	10
-	1	3	2	7
				3

Step 5: We can now subtract 2 tens from 9 tens we have left. We are left with 7 tens

Th	H	T	O
			

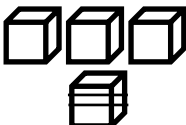


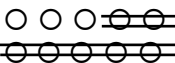
	Th	H	T	O
		4	9	
	4	5	10	10
-	1	3	2	7
			7	3

Step 6: We can now subtract 3 hundreds from 4 hundreds we have left. We are left with 1 hundred.

Th	H	T	O
			

	Th	H	T	O
		4	9	
	4	5	10	10
-	1	3	2	7
		1	7	3

Step 7: We can subtract 1 thousand from the 4 thousand that we have. We are left with 3 thousand.

Th	H	T	O
			

	Th	H	T	O
		4	9	
	4	5	10	10
-	1	3	2	7
	3	1	7	3

ANSWER: $4\ 500 - 1\ 327 = 3\ 173$

I was then planning on giving the participants 2 or 3 different examples involving “borrowing” and asking them to work in pairs describing what to do.